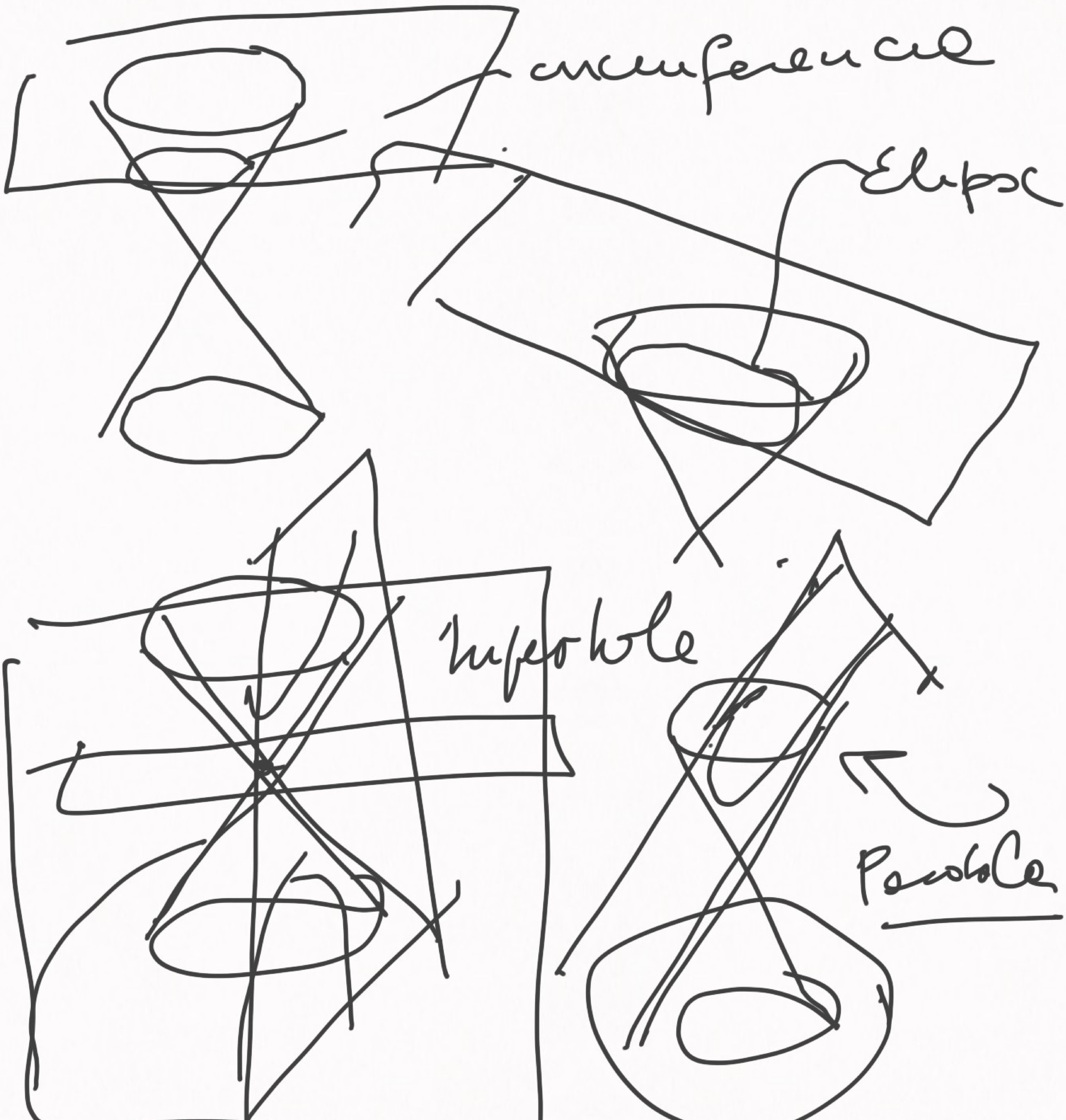
Conicas



conica duas retas vazio
deg. 1 ponto 2 pontos

⇓ Geometria Analitica

$$\Rightarrow \boxed{Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0} \Rightarrow \text{quadrática}$$

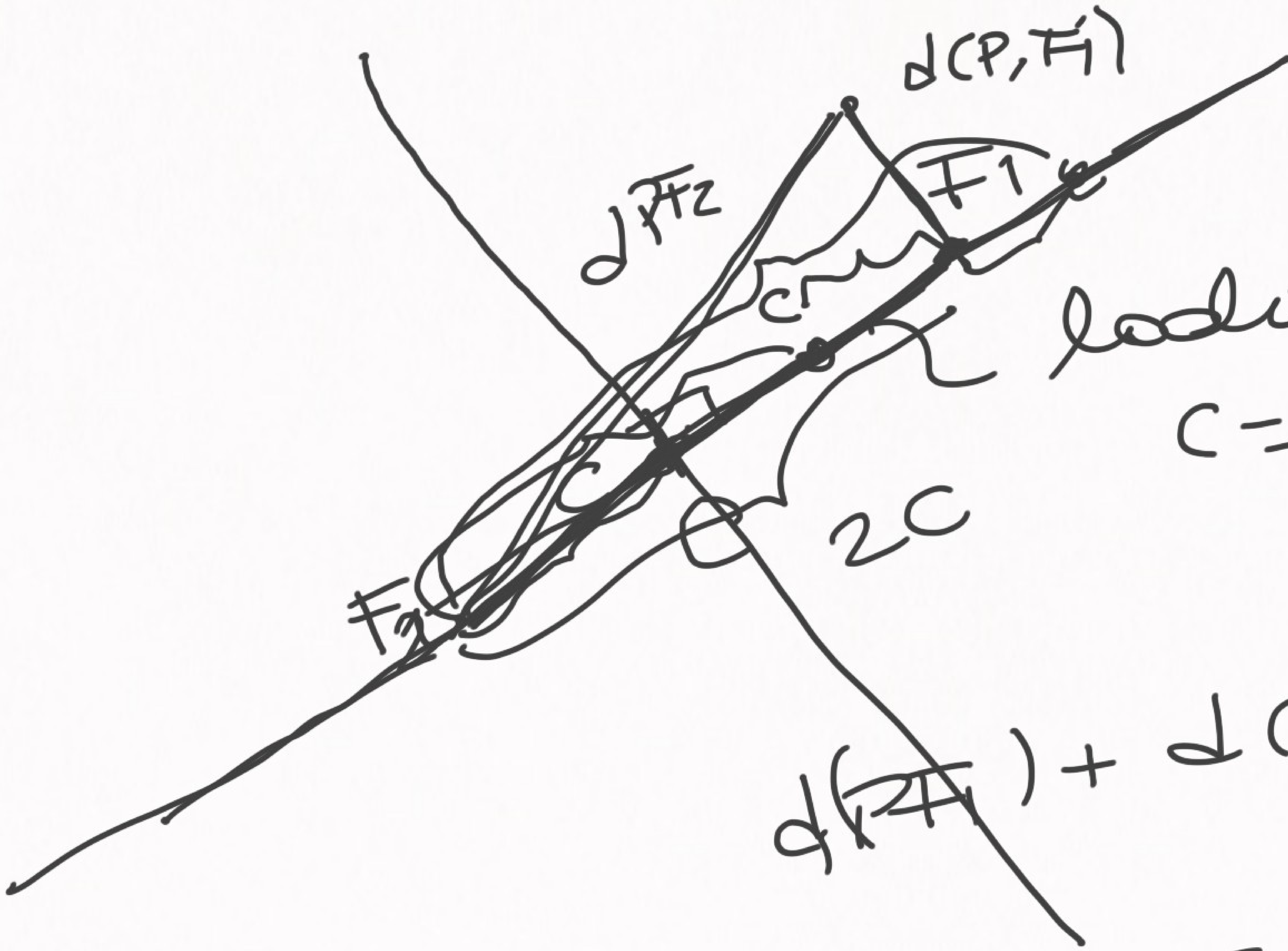
⇓

<u>Elipse</u>: dados dois focos F_1 e F_2, $d(F_1, F_2) = 2c$

a Elipse é o lugar geométrico dos pontos tal que

$$d(P,F_1) + d(P,F_2) = 2a$$

$$\boxed{a > c}$$



$$C = \frac{F_1 + F_2}{2}$$

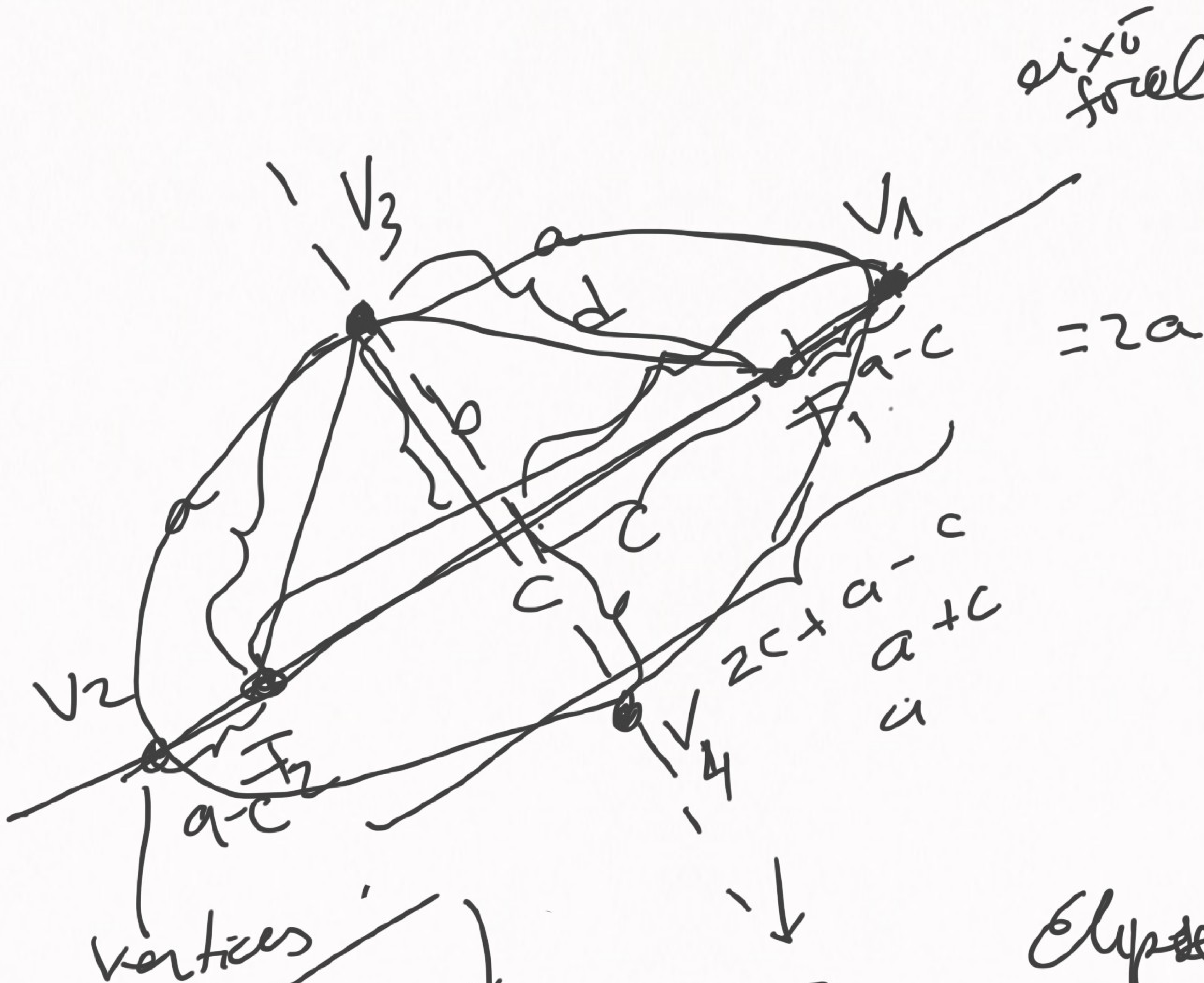
$$d(P,F_1) + d(P,F_2) =$$

$$2a < 2c$$

$\boxed{a>c}$ $\boxed{a<c}$

a soma do longitude de dois lado de um triangulo e sempre maior que o outro

importancia $a>c$



Elipse

$$a^2 = b^2 + c^2$$

$$b^2 = a^2 - c^2$$

medu

$$\boxed{e = \frac{c}{a} < 1}$$

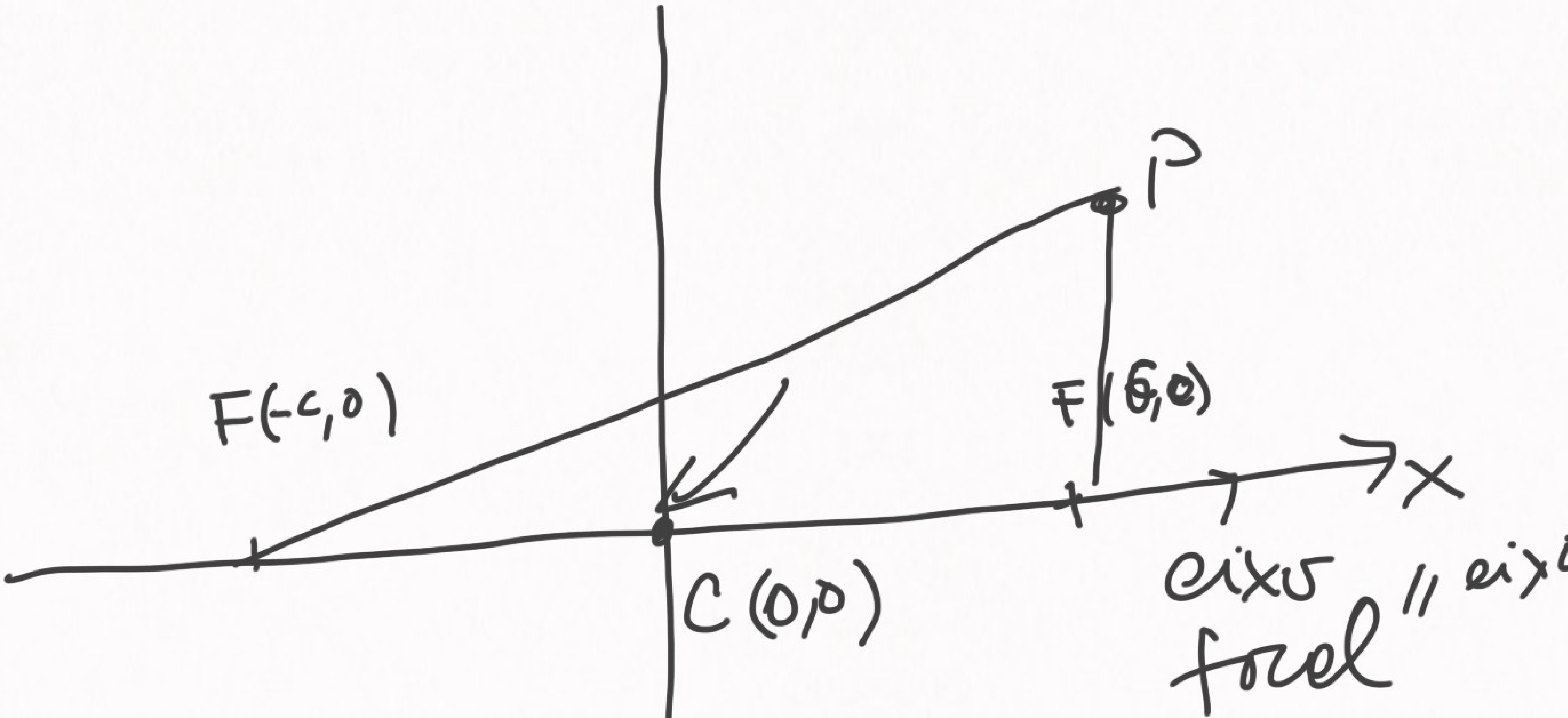
exentricidade da circunferencia e 0

Elementos do Elipse

- 4 vértices ✓
- eixo focal ✓
- centro ✓
- excentricidade ✓
- eixo não focal
- Focos

Elipse

$a > c$



$$d(P,F_1) + d(P,F_2) = 2a$$

$$\sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} = 2a$$

$$\left(\sqrt{(x-c)^2 + y^2}\right)^2 = \left(2a - \sqrt{(x+c)^2 + y^2}\right)^2$$

$$(x-c)^2 + \cancel{y^2} = 4a^2 - 4a\sqrt{(x+c)^2 + y^2}$$

$$= (x+c)^2 + \cancel{y^2}$$

$$\Longrightarrow$$

$$-\cancel{4}ac = \cancel{4}a^2 - \cancel{4}\sqrt{(x+c)^2+y^2}$$

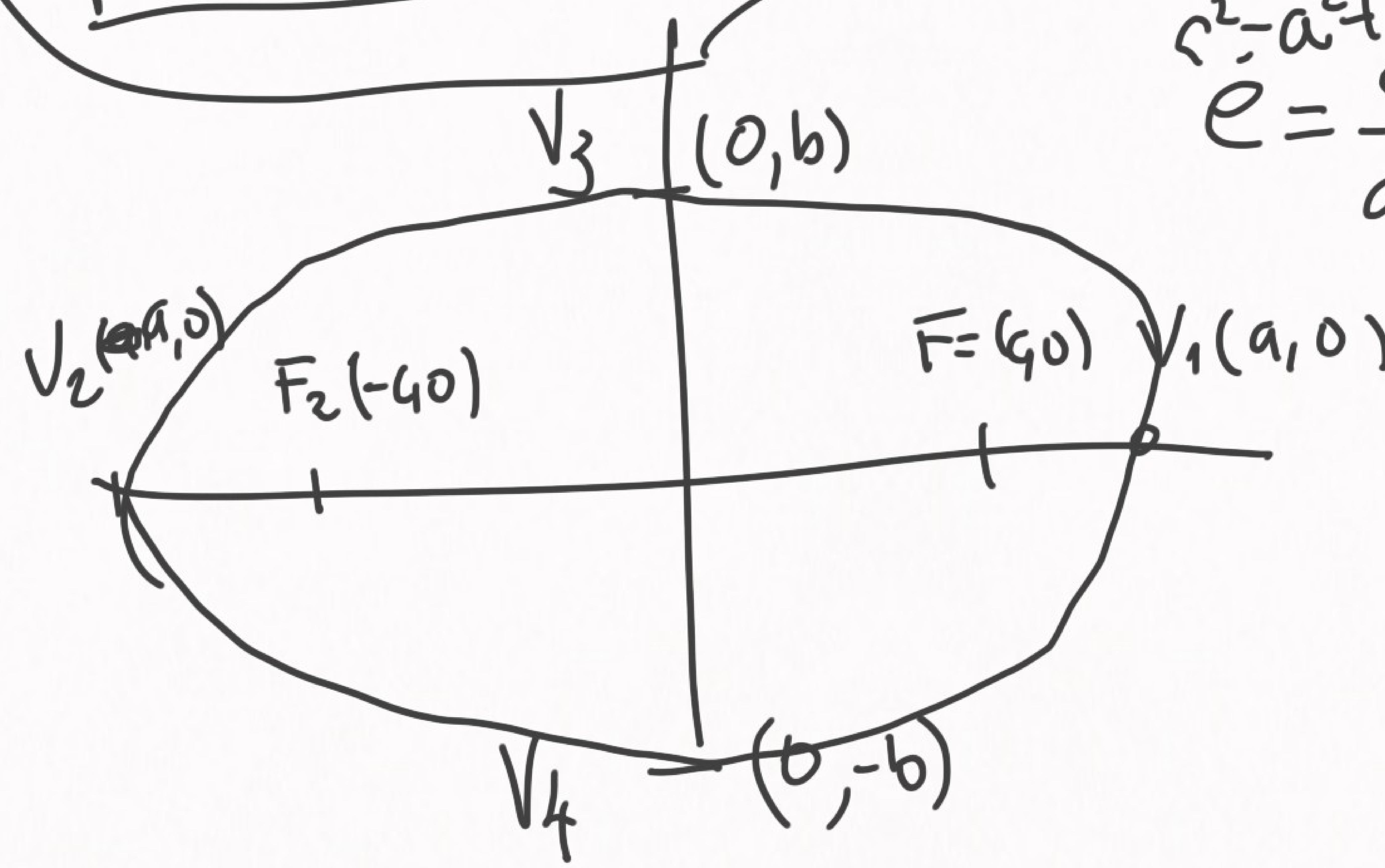
Exercício

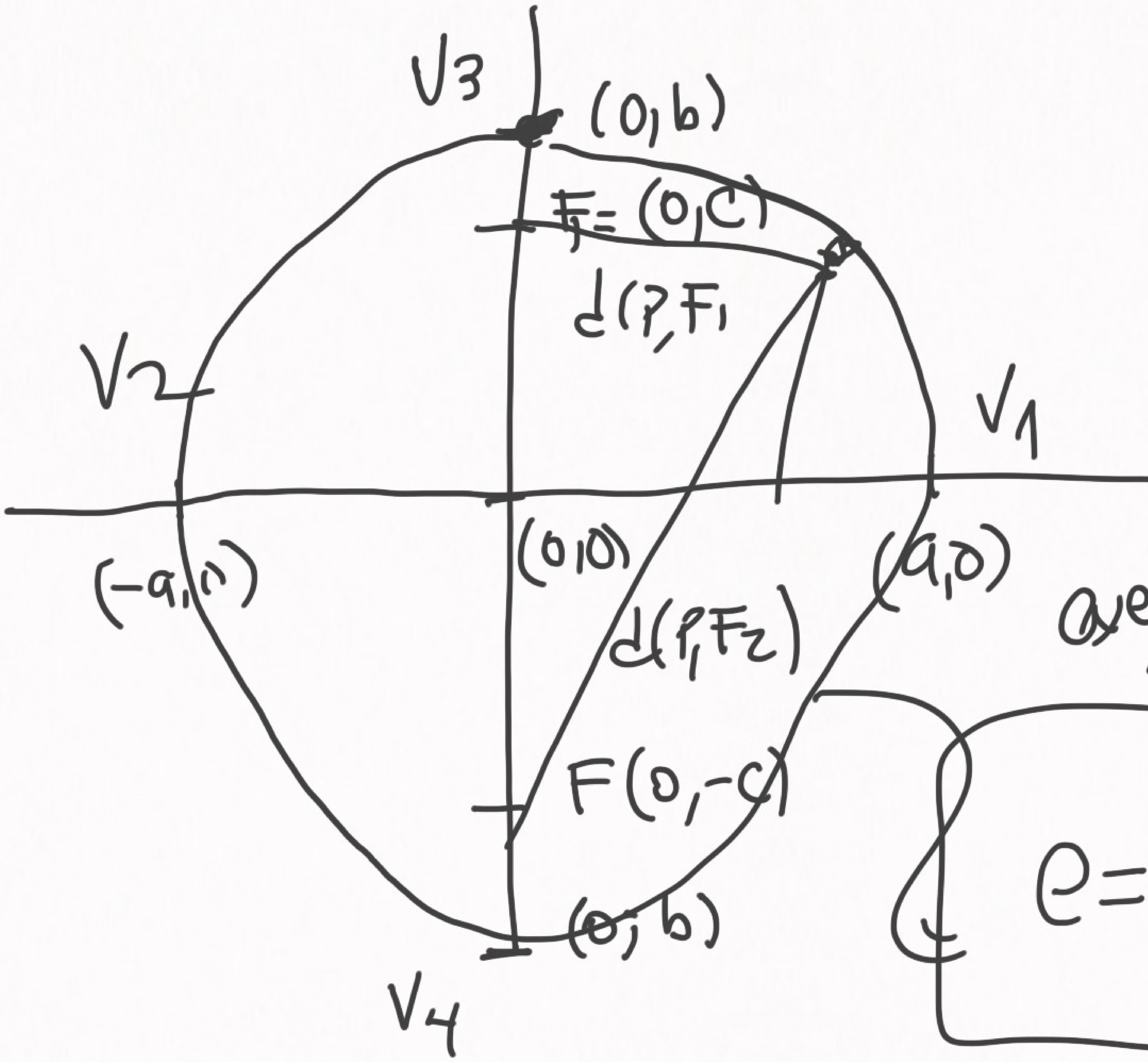
àpose b^2

$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}$$

$c^2 = a^2 + b^2$

$e = \frac{c}{a} < 1$





exercicio

$$e = \frac{b}{c}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b > a$$

$$b^2 = a^2 + c^2$$

quadratica

$\boxed{B=0}$

$$Ax^2 + \cancel{Bxy} + Cy^2 + Dx + Ey + F = 0$$

eixos focais // eixos cartesianos

A e $C \neq 0$

$sg(A) = sg(C)$ candidato a

Elipse

$$Ax^2 + Dx + Cy^2 + Ey + F = 0$$

$$A\left(x^2 + \frac{D}{A}x + \frac{D^2}{4A^2}\right) - \frac{D^2}{4A}$$

$$C\left(y^2 + \frac{E}{C}y + \frac{E^2}{4C^2}\right) - \frac{E^2}{4C} + F = 0$$

$$A\left(x+\frac{D}{2A}\right)^2+C\left(y+\frac{E}{2C}\right)^2=$$

$$\frac{D^2}{4A}+\frac{D^2}{4C}-F \sim r^2$$

$A>0$

$C>0$

>0

$$\frac{\left(y+\frac{D}{2A}\right)^2}{\left(\frac{r}{\sqrt{A}}\right)^2}+\frac{\left(y+\frac{E}{2C}\right)^2}{\left(\frac{r}{\sqrt{C}}\right)^2}=1$$

a^2 b^2

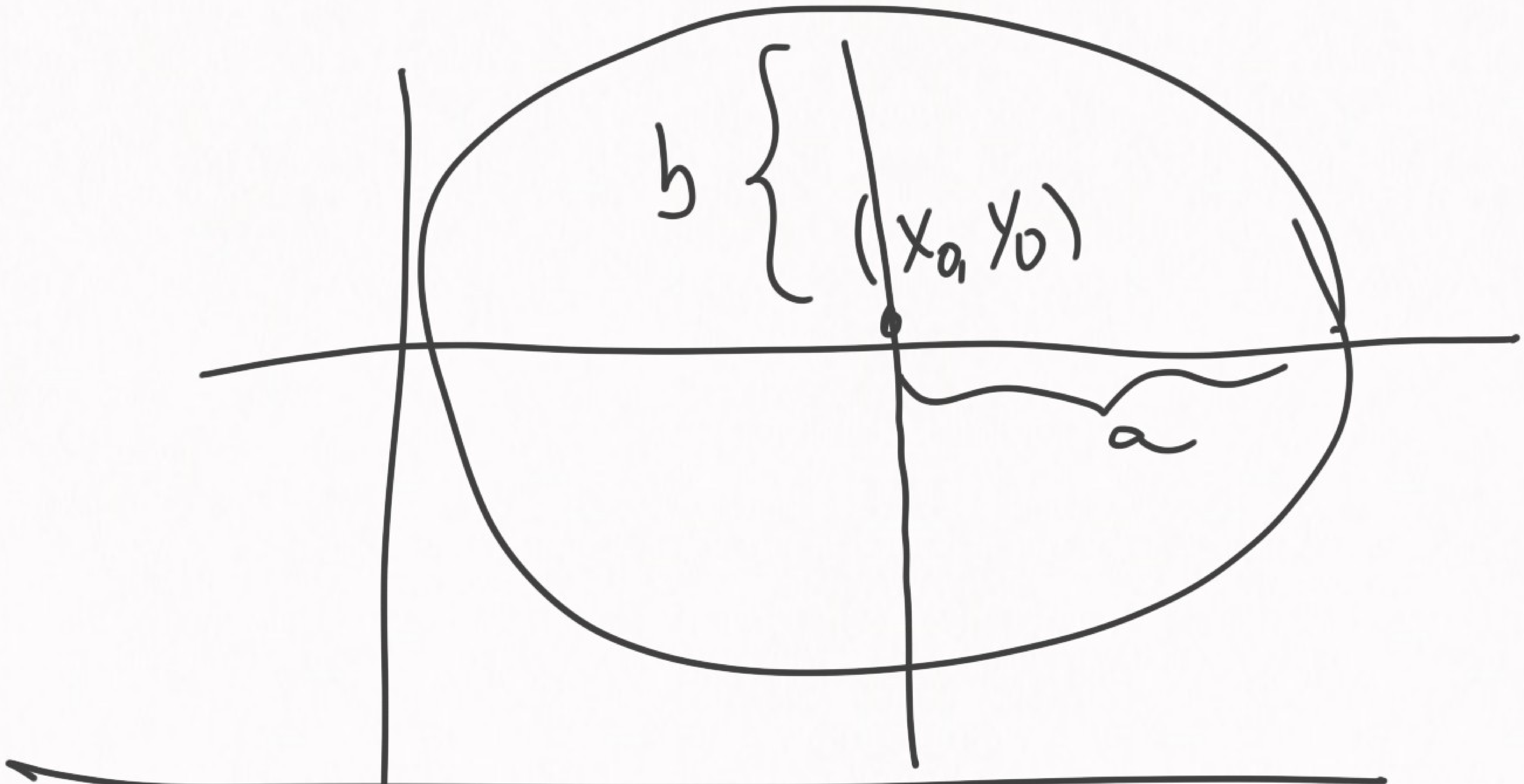
$$C=\left(-\frac{D}{2A},-\frac{E}{2C}\right)$$

$\Rightarrow c^2=a^2-b^2$

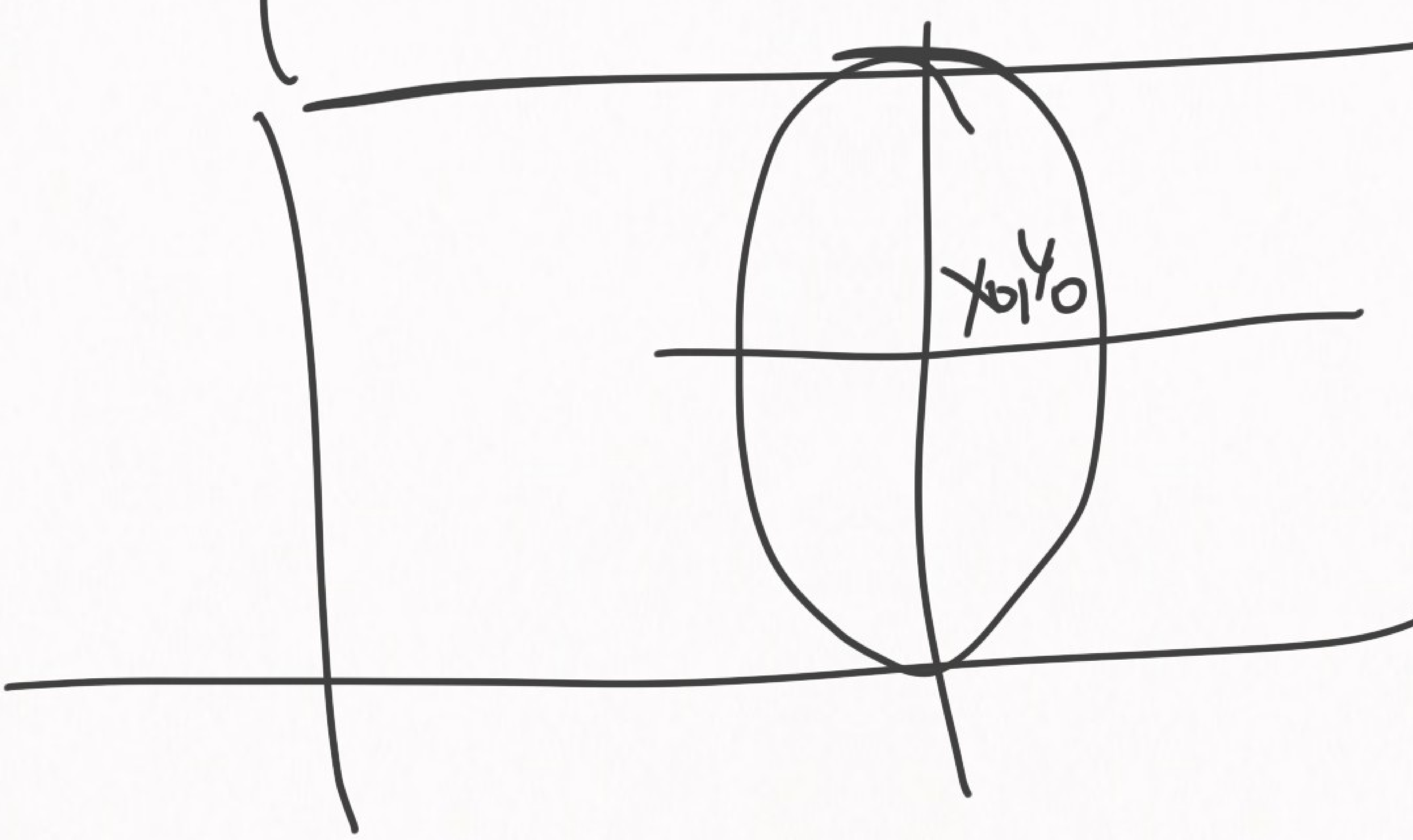
e

$a>b$

$a<b \rightarrow c^2=b^2-a^2$



$$\boxed{\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1}$$



Exemplo numerico

$$2x^2 + 6y^2 - 4x + 24y + F = 0$$

$$\boxed{8}$$

$$2(x^2 - 2x + 1) - 2$$

$$6(y^2 + 4y + 4) - 24 + 8 = 0$$

$$\frac{(x-1)^2}{9} + \frac{(y+2)^2}{3} = 1$$

$$\frac{(x-1)^2}{(3)^2} + \frac{(y+2)^2}{(\sqrt{3})^2} = 1$$

$a = 3$ $\qquad$ $b = \sqrt{3}$

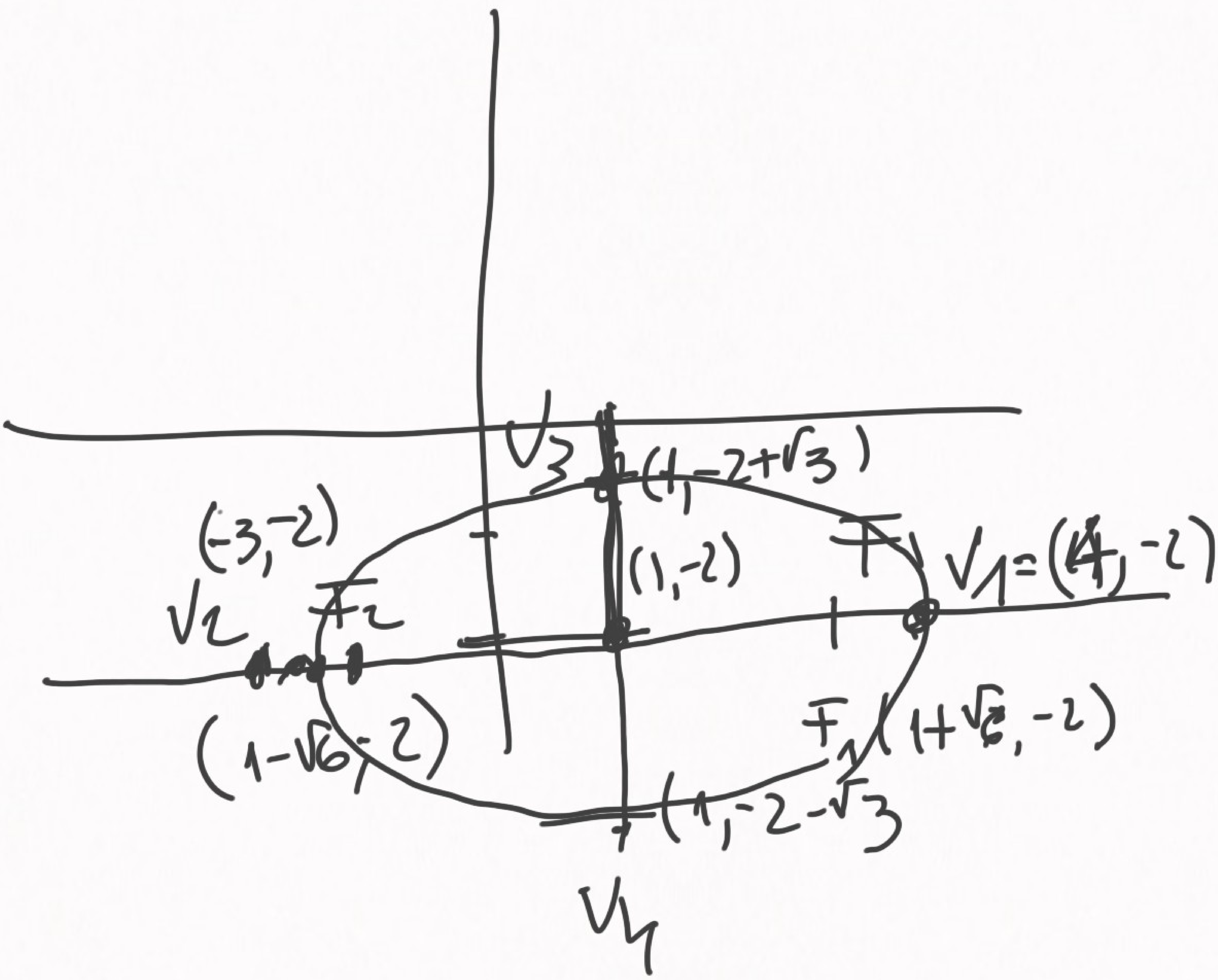
$a > b$ $\qquad$ $e = \frac{\sqrt{6}}{3}$

eixo focal // eixo x

$$c = \sqrt{\underset{a^2}{9} - \underset{b^2}{3}} = \sqrt{6}$$

$C = (1, -2)$

$F =$

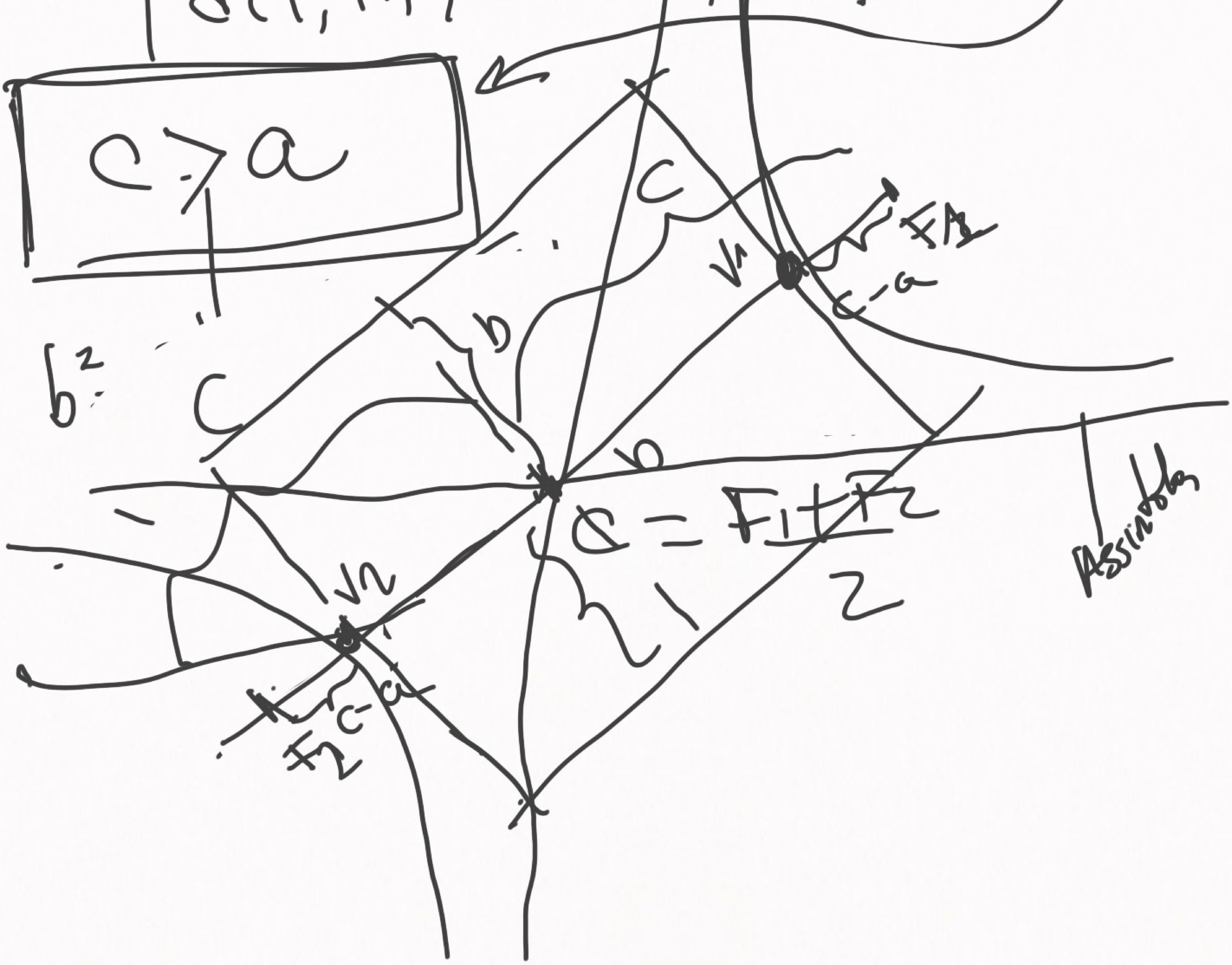


Hiperboles

<u>definição</u> dados dois focos F_1, F_2 $d(F_1, F_2) = 2c$, uma hiperbole é o lugar geométrico dos pontos P tal que

$$|d(P, F_1) - d(P, F_2)| = 2a$$

$$\boxed{c > a}$$



$\therefore$

dois Focos F_1, F_2

dois vértices V_1, V_2

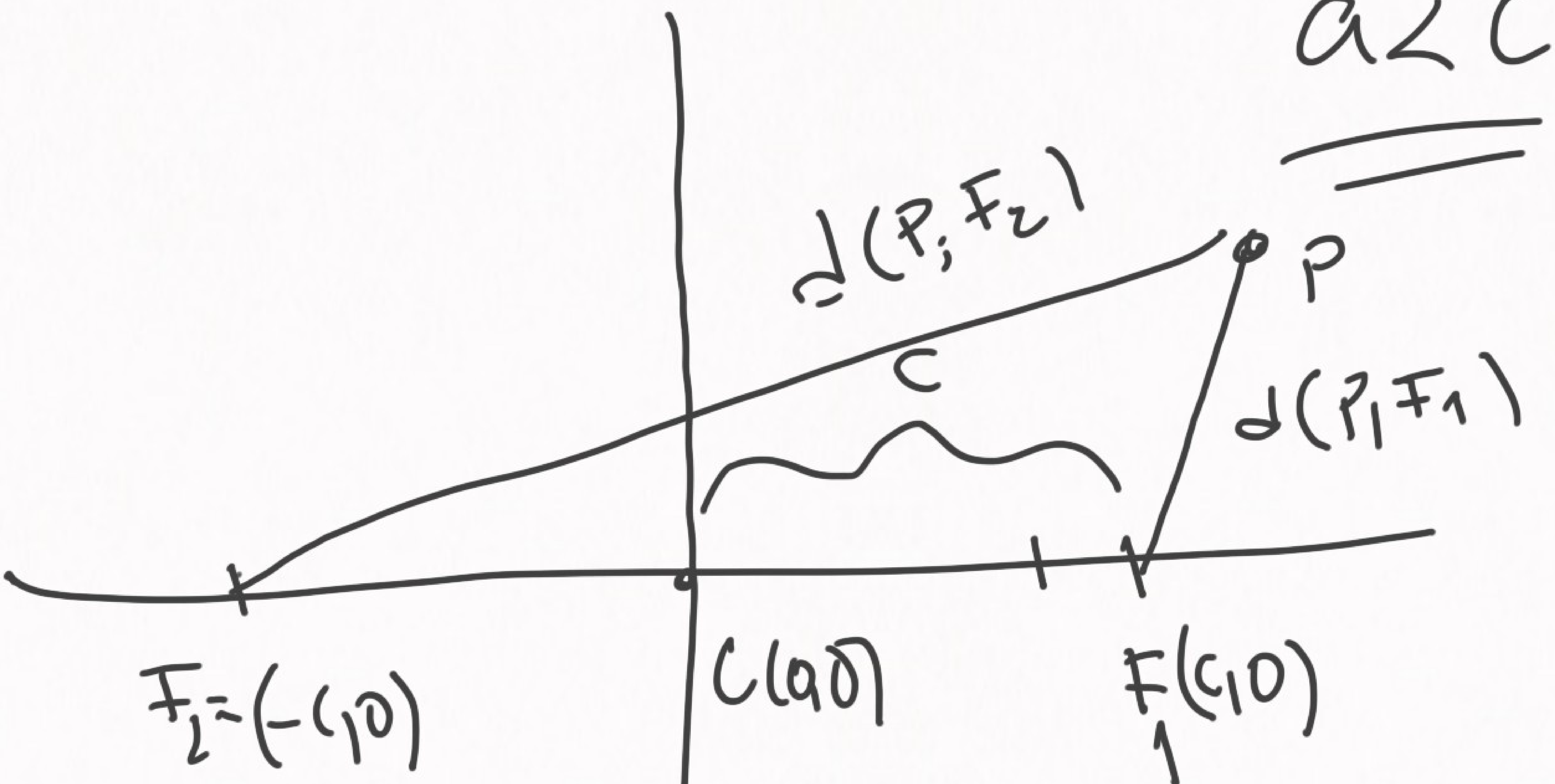
duas Assíntotas

a excentricidade $e > 1$

eixo focal

eixo não focal

$a < c$



$$\left|\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2}\right|=2a$$

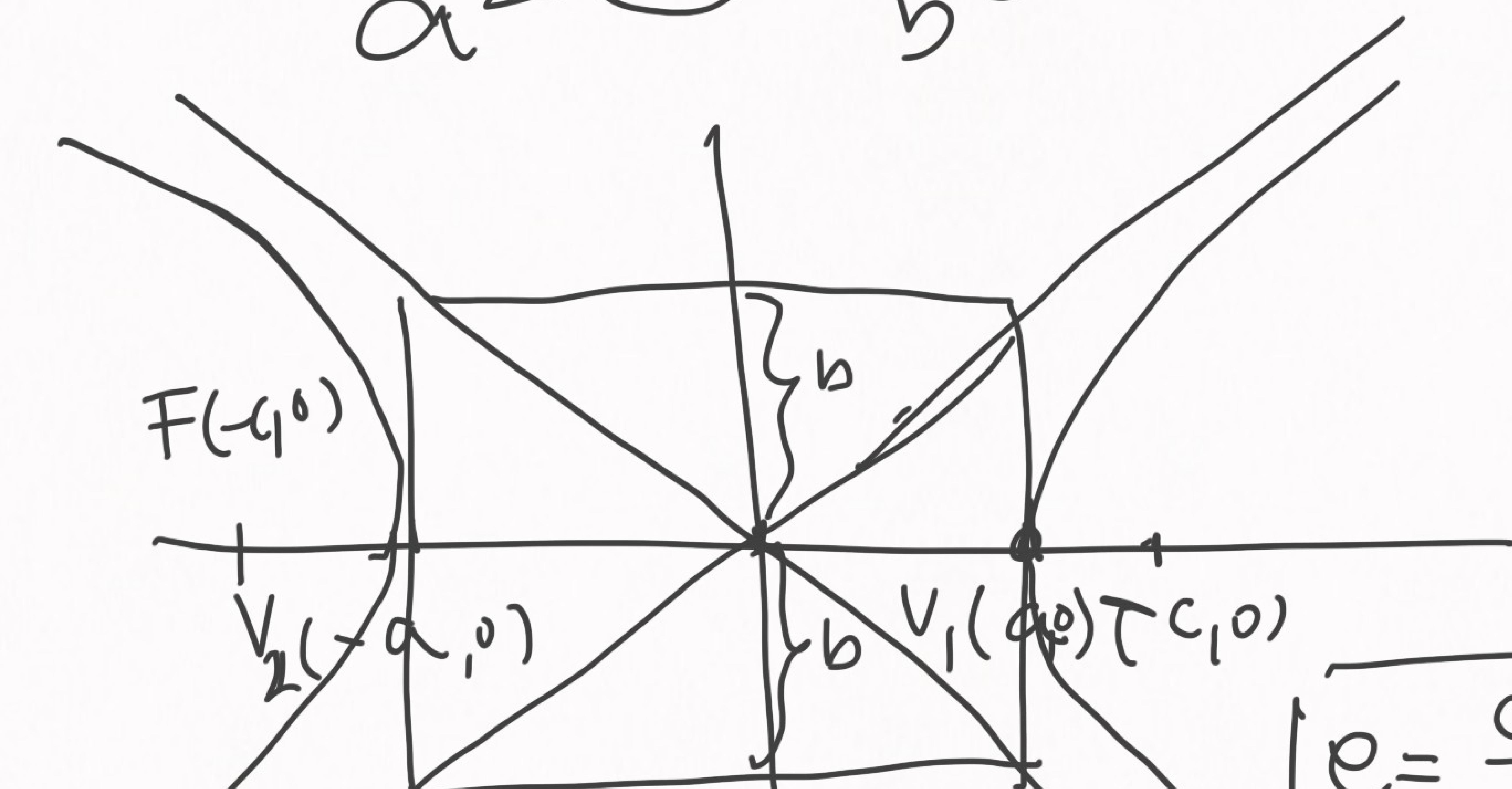
$$\left(\sqrt{(x+c)^2+y^2}\right)^2=\left(\pm 2a+\sqrt{(x-c)^2+y^2}\right)^2$$

$\rightsquigarrow$ deux cases

$$(x+c)^2+y^2=4a^2\pm 4a\sqrt{(x-c)^2+y^2}+$$
$$+(x-c)^2+y^2 \Longrightarrow$$

Exercício

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



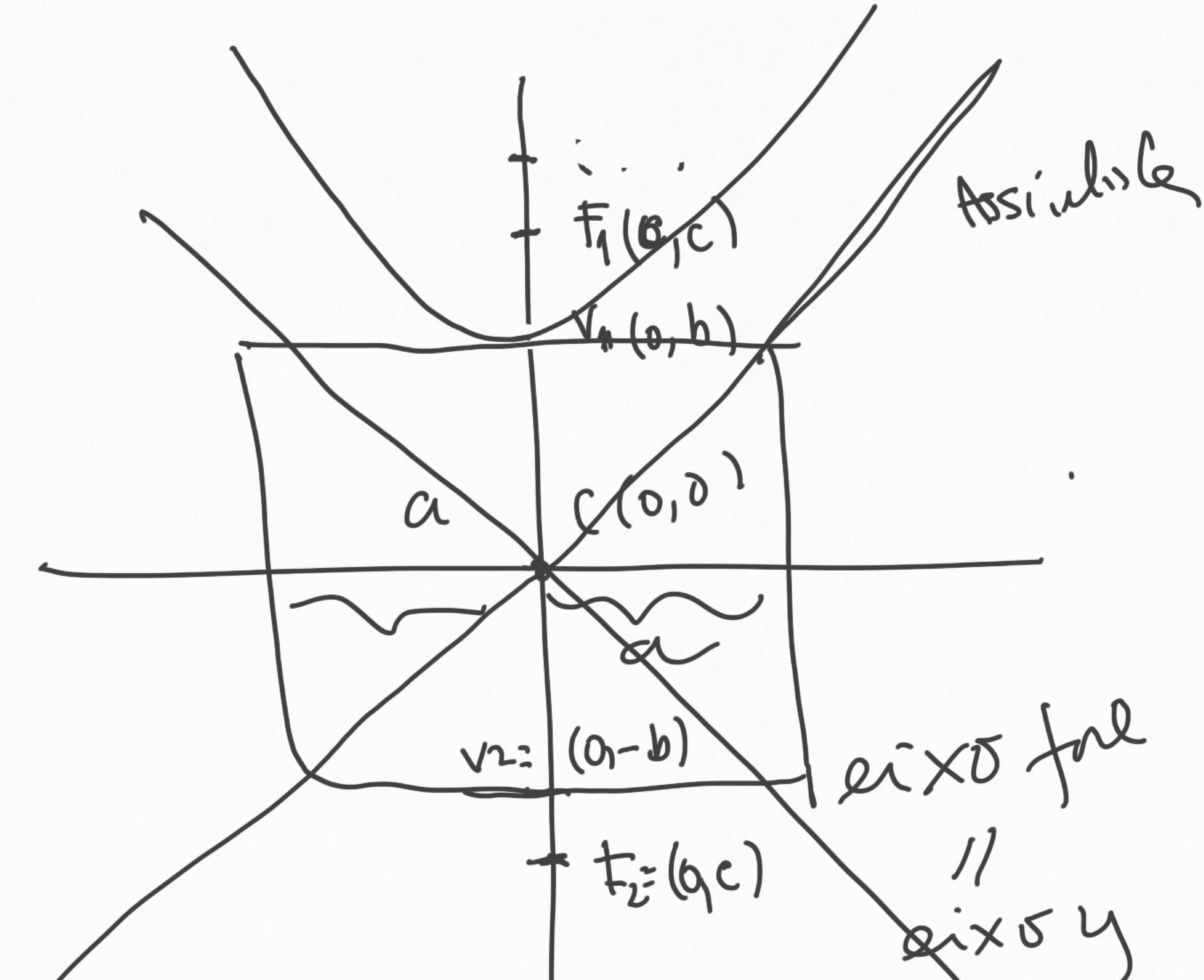
$$e = \frac{c}{a} > 1$$

$c^2 = a^2 + b^2$

$F_1 = (c,0)$, $F_2 = (-c,0)$

$V_1 = (a,0)$ $V_2 = (-a,0)$

Assíntotas $y = \frac{b}{a}x$ $y = -\frac{b}{a}x$



$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad c^2=a^2+b^2$$

$$\text{Assi } y=\frac{b}{a} \qquad y=-b/a$$

1

$$\boxed{e=\frac{c}{b}>1}$$

$$2x^2 - y^2 + 4x + 2y + \overset{=5}{F} = 0$$

$$2(x^2+4x+4) - 8 - [y^2 - 2y + 1] + 1 + 5 = 0$$

$$2(x+2)^2 - (y-1)^2 = 2$$

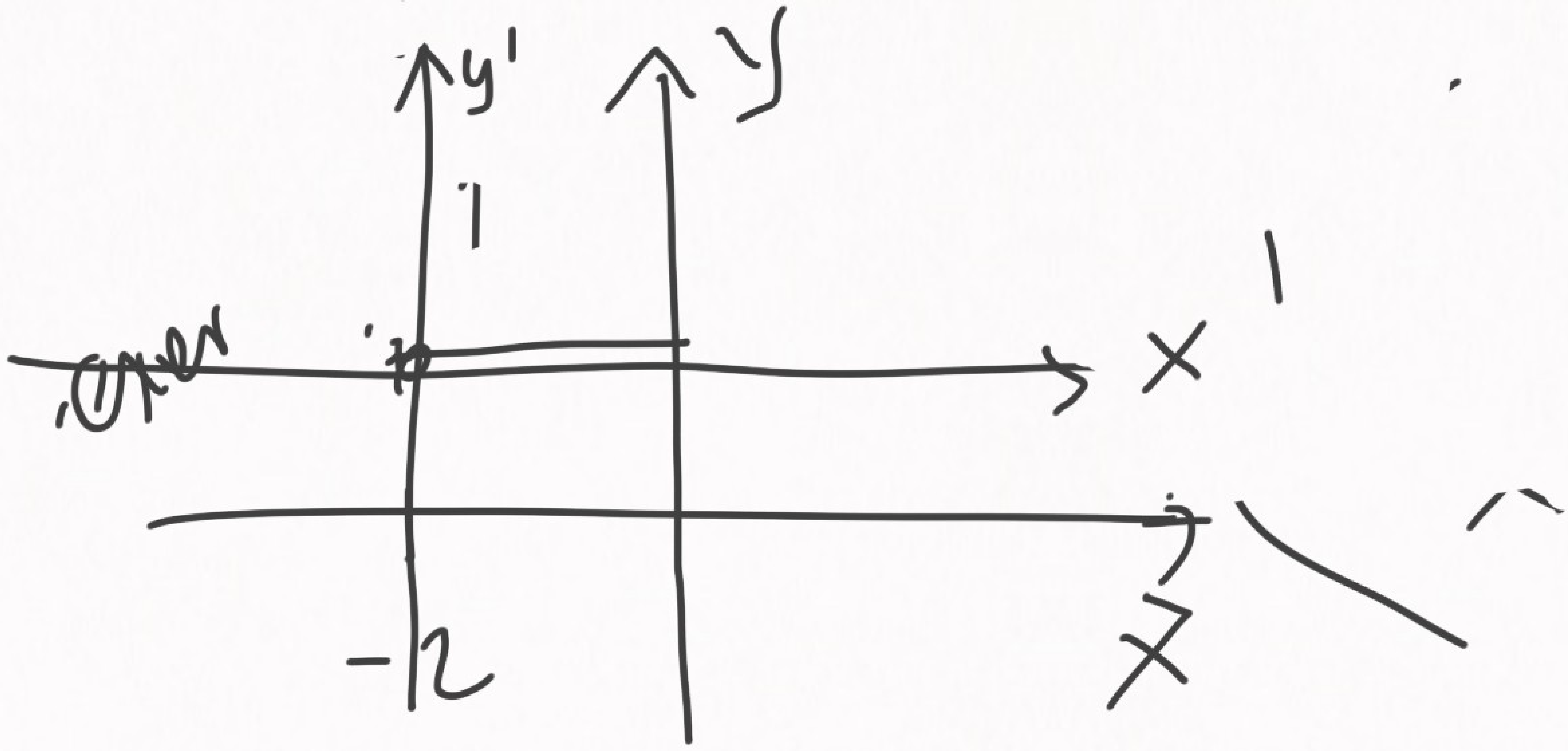
$$\boxed{(x+2)^2 - \frac{(y-1)^2}{(\sqrt{2})^2} = 1}$$

$C = (-2, 1)$ $\quad a = 1 \quad b = \sqrt{2}$

$e = \frac{1}{\sqrt{3}} \quad c = \sqrt{1^2 + (\sqrt{2})^2} = \sqrt{3}$

$x'^2 - \frac{y'^2}{(\sqrt{2})^2} = 1$

$$y - 1 = \sqrt{2}(x+2)$$
$$y - 1 = -\sqrt{2}(x+2)$$



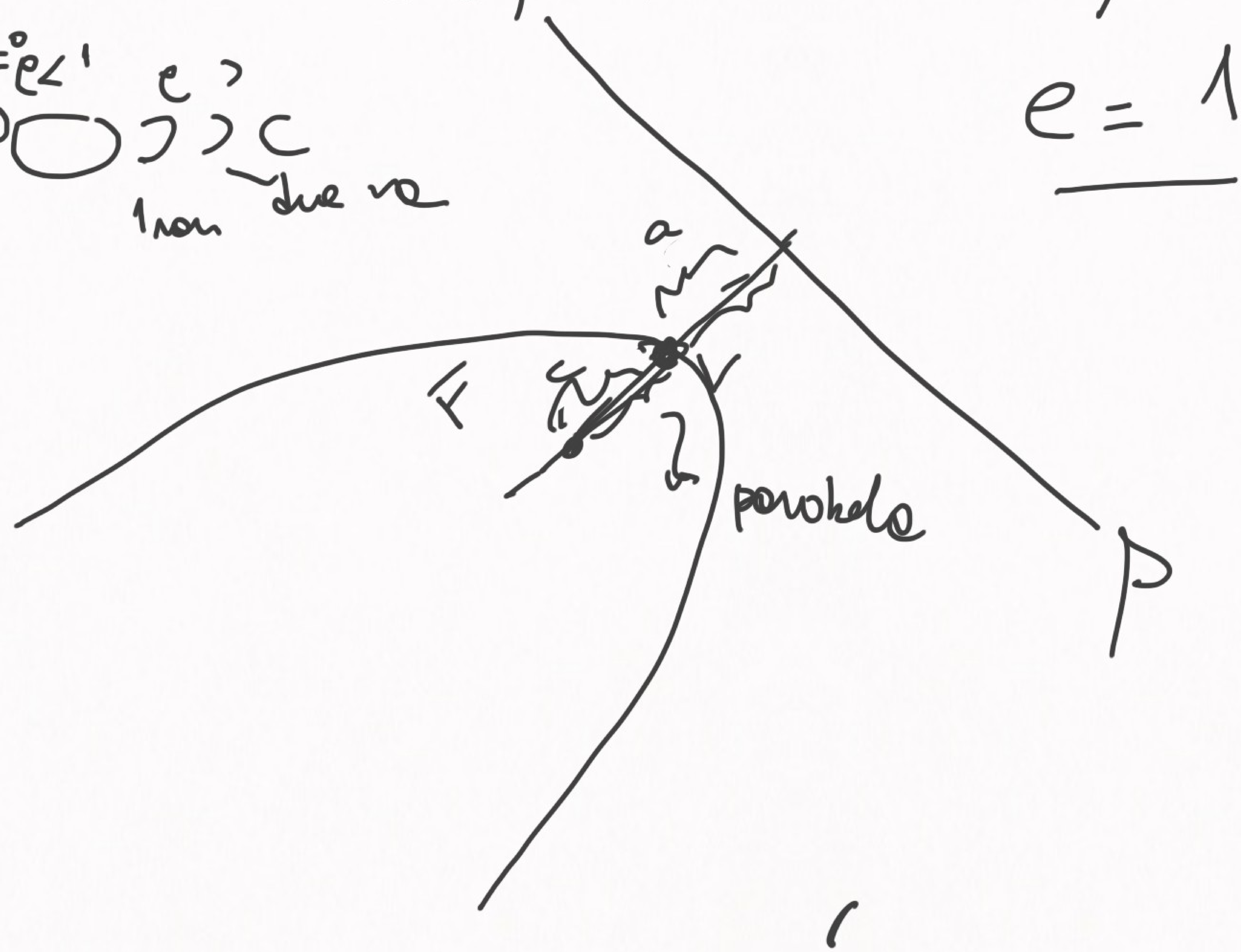
$$\frac{(x+2)^2}{} - \frac{(y-1)^2}{(\sqrt{2})^2} = \frac{x'^2}{} - \frac{y'^2}{(\sqrt{2})^2} = 1$$

Parabolas

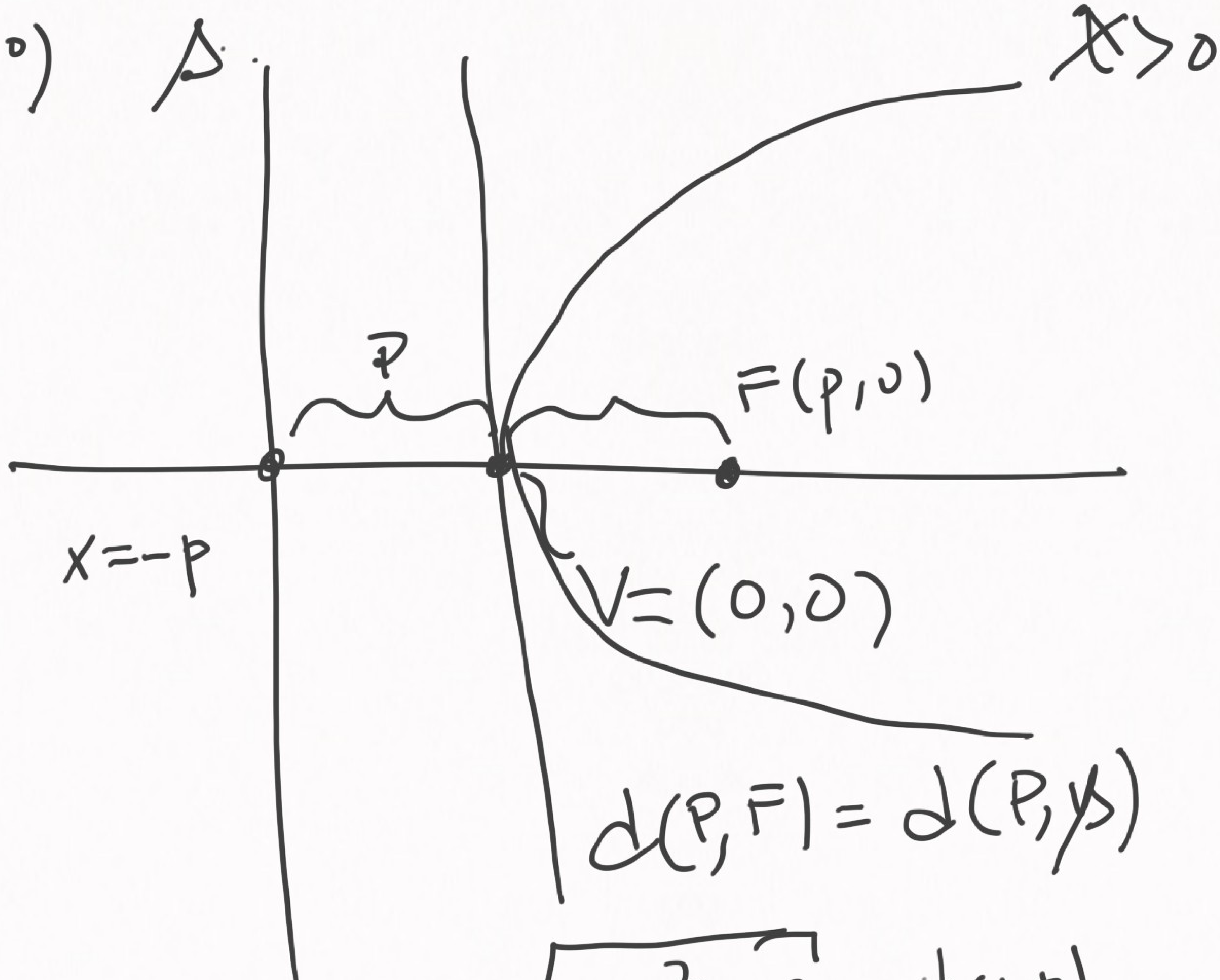
definição: dado um Foco F e uma diretriz Δ, a parabola é o lugar geometrico dos pontos P tal que

$$d(P,F) = d(F,\Delta)$$

$e = 1$



1°)



$$d(P,F) = d(P,\Delta)$$

$$\sqrt{(x-p)^2+y^2} = |x+p|$$

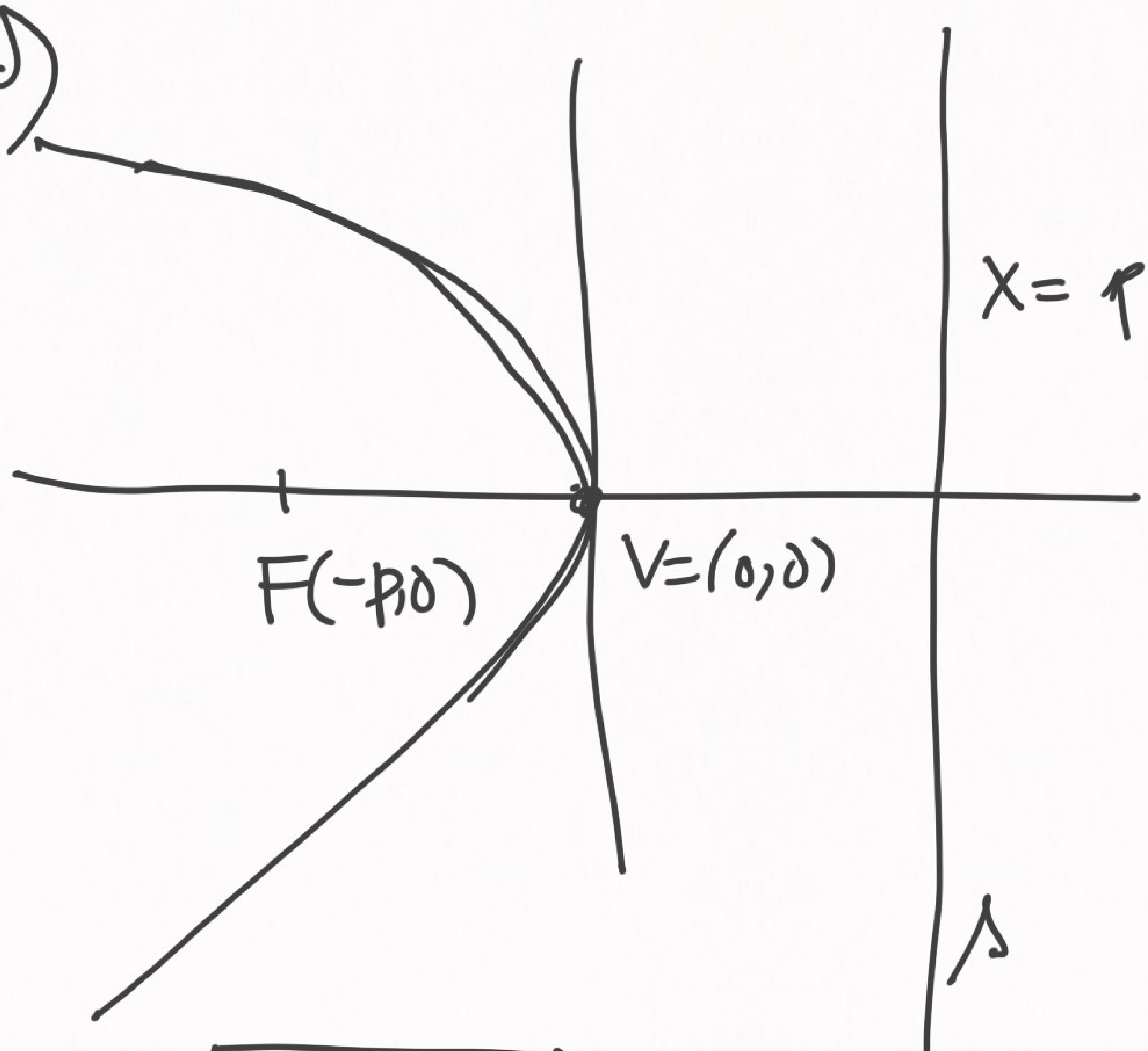
$$(x-p)^2+y^2=(x+p)^2$$

$$\cancel{x^2}-2xp+\cancel{p^2}+y^2=\cancel{x^2}+2xp+\cancel{p^2}$$

$$\boxed{y^2=4px}$$

:

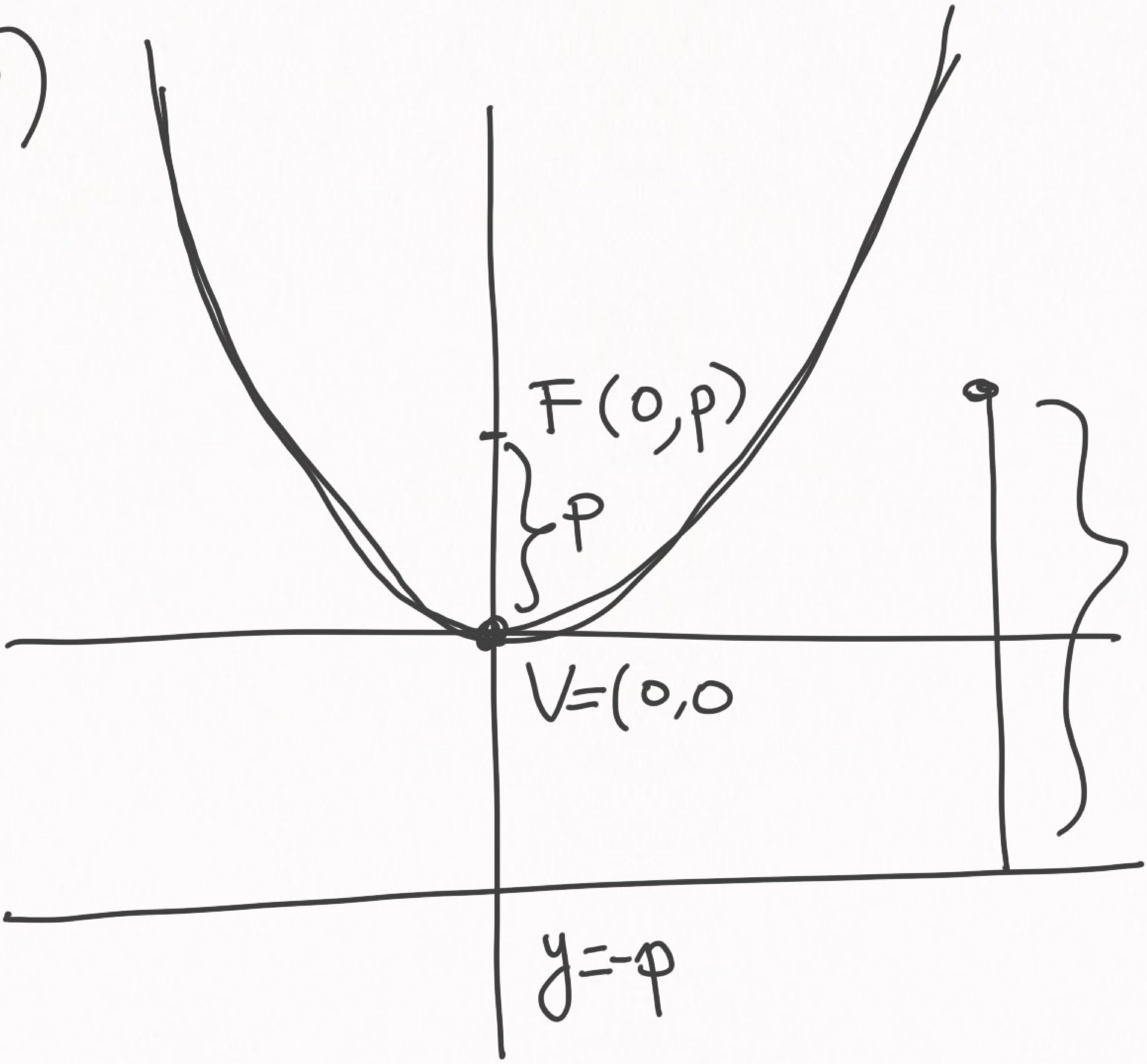
2d)



$$\sqrt{(x+p)^2+y^2} = |x-p|$$

$$\Rightarrow \boxed{y^2 = -4px}$$

3º)

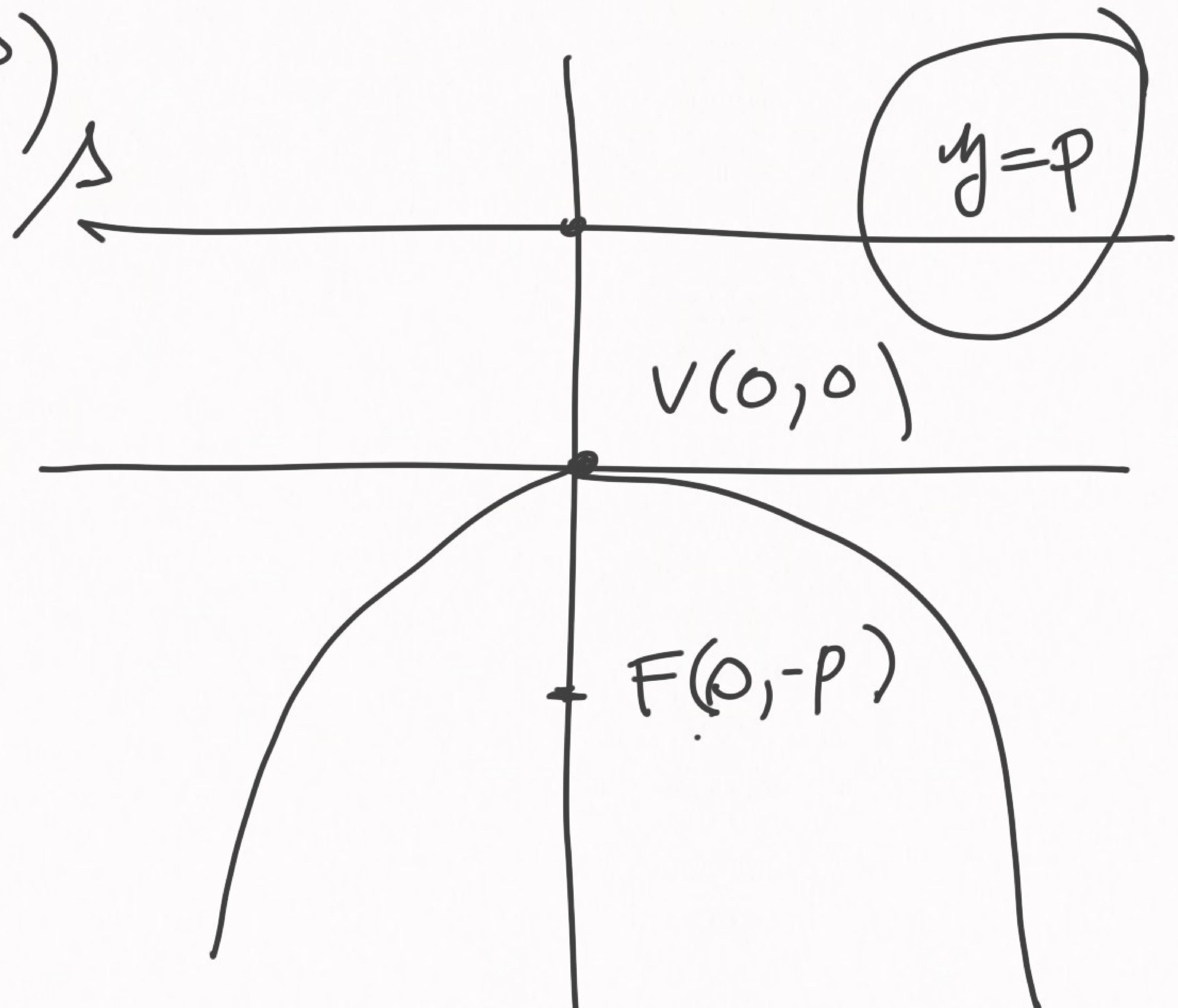


$$\sqrt{x^2+(y-p)^2} = |y+p|$$

$$\hookrightarrow \boxed{x^2 = 4py}$$

+

4°)



$$\sqrt{(y+p)^2+x^2} = |y-p|$$

$$\rightsquigarrow \boxed{x^2 = -4py}$$

$$y^2 - 3x + 4y + 2 = 0$$

$\underbrace{C \neq 0 \quad A = 0}$ $\longrightarrow$ cod
a parabola

$$\underbrace{y^2 + 4y + 4} - 4 - 3x + 2 = 0$$

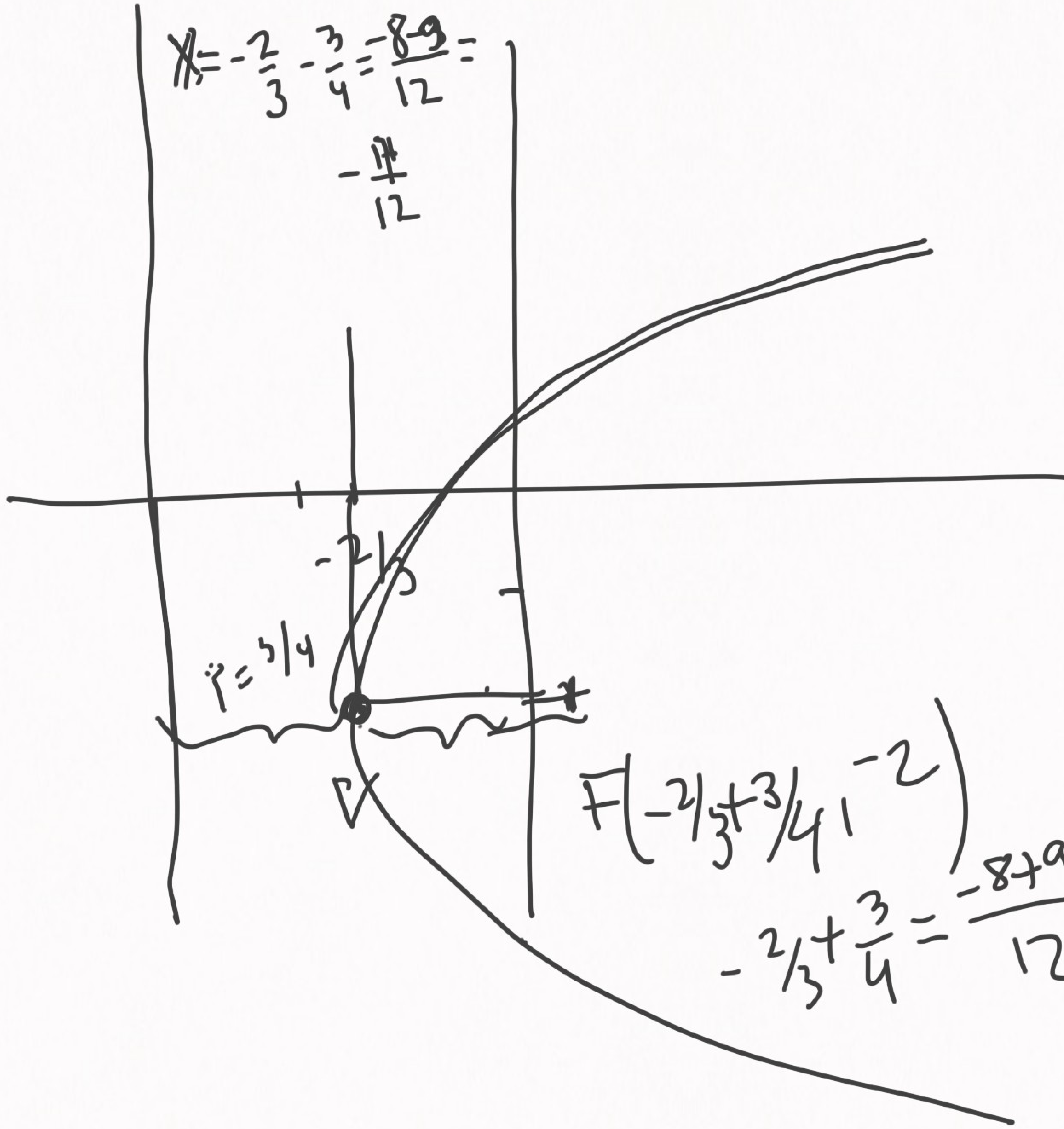
$$(y+2)^2 = 3x + 2$$

$$(y+2)^2 = 3\left(x + \frac{2}{3}\right)$$

$$(y+2)^2 = 4\left(\frac{3}{4}\right)\left(x + 2/3\right)$$

$p = \boxed{\frac{3}{4}}$ $\quad V = \left(-2/3, -2\right)$ $\quad F = \left(\frac{1}{12}, -2\right)$

$x = -\frac{2}{3} - \frac{3}{4} = \frac{-8-9}{12} = -\frac{17}{12}$



$F\left(-\frac{2}{3}+\frac{3}{4}, -2\right)$

$-\frac{2}{3}+\frac{3}{4} = \frac{-8+9}{12} = \frac{1}{12}$

$$Ax^2 + \cancel{Bxy} + Cy^2 + Dx + Ey + F = 0$$

$A \neq 0$ e $C \neq 0$ $\quad A \neq C$

$sig(A) = sig(C) \Rightarrow$ cu Elipse

$sig(A) \neq sig(C) \Rightarrow$ " Hyperbole

$A = 0, C \neq 0 \Rightarrow$ cond a Parabola

$A \neq 0 \quad C = 0$ " "